

## ΑΠΑΝΤΗΣΕΙΣ

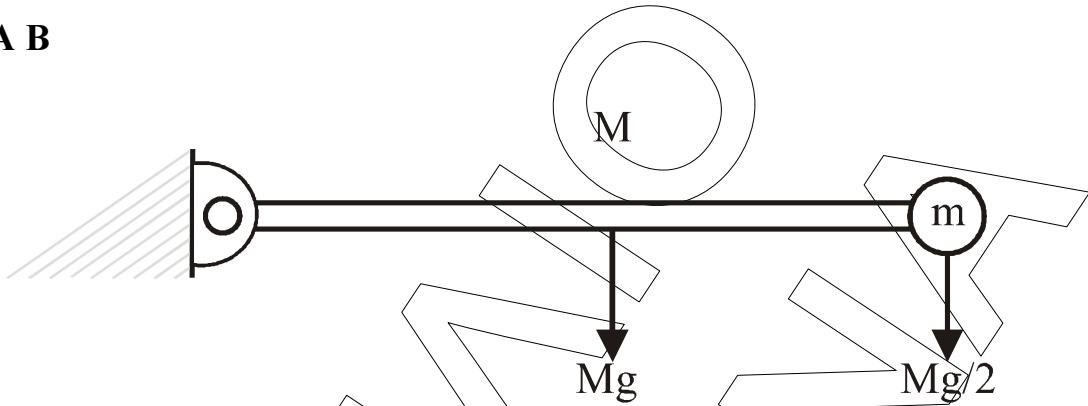
### ΘΕΜΑ Α

A1. → α)    A2. → β)    A3. → α)    A4. → δ)

A5.    α) → Λάθος    β) → Σωστό    γ) → Σωστό    δ) → Λάθος    ε) → Σωστό

### ΘΕΜΑ Β

B1.



Για σύστημα ράβδου - μάζας

$$\Sigma \tau = I_p \cdot \alpha_{\gamma\omega\nu} \Rightarrow \frac{M}{2} g \cdot L + Mg \frac{L}{2} = \left( \frac{1}{3} ML^2 + \frac{M}{2} L^2 \right) \alpha_{\gamma\omega\nu} \Rightarrow$$

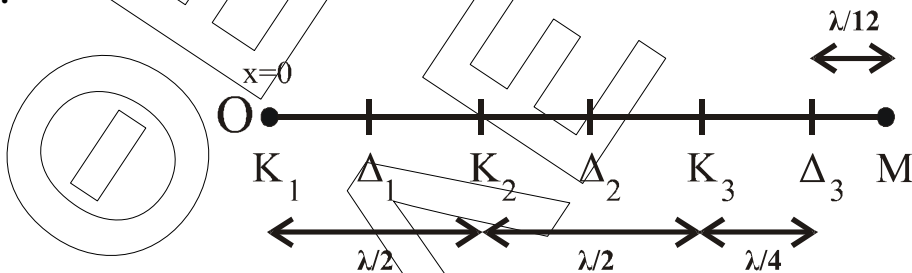
$$\Rightarrow M \cdot g \cdot L = \frac{5}{6} M \cdot L^2 \cdot \alpha_{\gamma\omega\nu} \Rightarrow \alpha_{\gamma\omega\nu} = \frac{6g}{5L}.$$

Για τη ράβδο

$$\frac{\Delta L}{\Delta t} = \Sigma \tau_{\rho\alpha\beta} = I_{\rho\alpha\beta} \cdot \alpha_{\gamma\omega\nu} = \frac{1}{3} ML^2 \cdot \frac{6g}{5L} = \frac{2}{5} MgL.$$

Άρα σωστό το iii.

B2.



Η απόσταση του M από το O (αρχή  $x = 0$ ):

$$x_M = 2 \frac{\lambda}{2} + \frac{\lambda}{4} + \frac{\lambda}{12} = \frac{16}{12} \lambda = \frac{4}{3} \lambda$$

Το πλάτος:

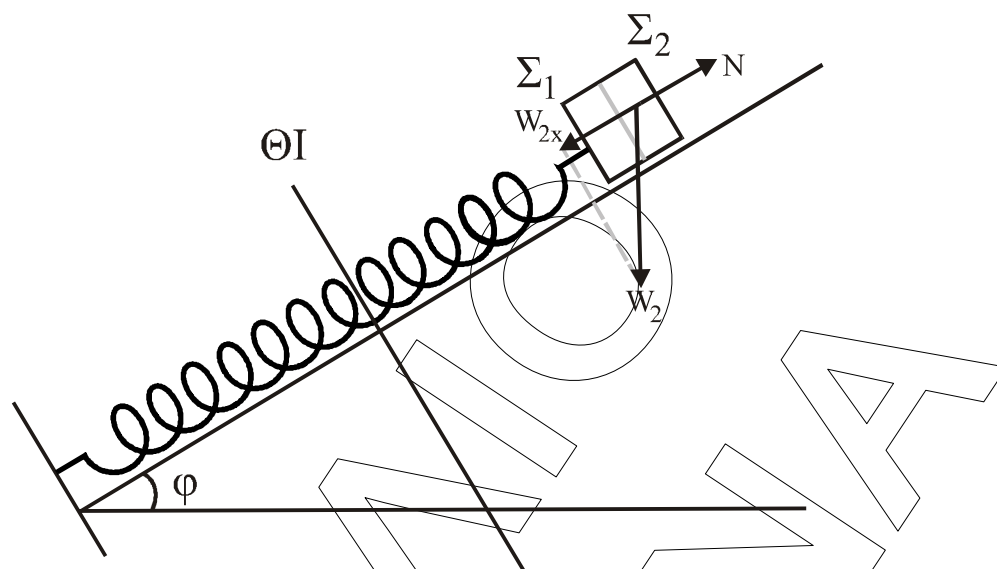
$$A' = \left| 2A \cdot \sin \frac{2\pi x_M}{\lambda} \right| = \left| 2A \cdot \sin \frac{2\pi \cdot 4\lambda}{3\lambda} \right| \Rightarrow A' = \left| 2A \cdot \sin \frac{8\pi}{3} \right| = \left| 2A \cdot \sin \left( \frac{6\pi + 2\pi}{3} \right) \right| =$$

$$= \left| 2A \cdot \sin \left( \frac{2\pi}{3} \right) \right| = \left| -2A \cdot \frac{1}{2} \right| = A.$$

Άρα το πλάτος είναι  $A' = A$ .

Άρα σωστή η (iii).

B3.



$$W_{2x} = m_2 g \eta \mu \phi$$

$$N - W_{2x} = -D_2 \cdot x$$

$$N = m_2 g \eta \mu \phi - D_2 \cdot x$$

Για να μην αποχωριστεί  $N > 0$

$$m_2 g \eta \mu \phi - D_2 x > 0$$

$$x < \frac{m_2 g \eta \mu \phi}{D_2} \quad (1)$$

$$\left. \begin{aligned} D_2 &= m_2 \omega^2 \\ k &= (m_1 + m_2) \omega^2 \end{aligned} \right\}$$

$$D_2 = \frac{m_2}{m_1 + m_2} k$$

$$D_2 = \frac{m_2}{m_1 + m_2} k \quad (2)$$

$$\text{Από (1) και (2)} \quad x < \frac{m_2 \cdot g \cdot \eta \mu \phi}{\frac{m_2 \cdot k}{m_1 + m_2}} \quad \text{ή} \quad x < \frac{(m_1 + m_2)}{k} g \cdot \eta \mu \phi$$

$$\text{ή} \quad kx \leq (m_1 + m_2) g \eta \mu \phi$$

$$\text{για } x = A, \quad kA \leq (m_1 + m_2) g \eta \mu \phi.$$

Άρα σωστή η (i).

## ΘΕΜΑ Γ

Από ΑΔΕΗΤ

$$E = U_E + U_B \Rightarrow U_E = E - U_B \Rightarrow U_E = E - \frac{Li^2}{2}$$

$$\text{Δίνεται: } U_E = 8 \cdot 10^{-2} (1 - i^2) = 8 \cdot 10^{-2} - 8 \cdot 10^{-2} \cdot i^2$$

$$\text{Με σύγκριση } E = 8 \cdot 10^{-2} \text{ J}$$

$$\frac{L}{2} = 8 \cdot 10^{-2} \Rightarrow L = 16 \cdot 10^{-2} \text{ H.}$$

$$E = \frac{C \cdot V_{\max}^2}{2} \Rightarrow 8 \cdot 10^{-2} = \frac{C \cdot 40^2}{2} \Rightarrow C = 10^{-4} \text{ F.}$$

$$\Gamma 1. \quad T = 2\pi \cdot \sqrt{LC} = 2\pi \cdot \sqrt{16 \cdot 10^{-2} \cdot 10^{-4}} = 2\pi \cdot 4 \cdot 10^{-3} \Rightarrow T = 8\pi \cdot 10^{-3} \text{ s.}$$

$$\Gamma 2. \quad U_E = \frac{q^2}{2C} = \frac{Q^2}{2C} \cdot \sigma \nu \nu^2 \omega t \Rightarrow U_E = E \cdot \sigma \nu \nu^2 \omega t.$$

$$\text{Για } t = \frac{T}{12} \text{ και } E = 8 \cdot 10^{-2} \text{ J, έχουμε:}$$

$$U_E = 8 \cdot 10^{-2} \cdot \sigma \nu \nu^2 \frac{\omega T}{12} \Rightarrow U_E = 8 \cdot 10^{-2} \cdot \sigma \nu \nu^2 \frac{\pi}{6} = 8 \cdot 10^{-2} \cdot \left(\frac{\sqrt{3}}{2}\right)^2 \Rightarrow$$

$$\Rightarrow U_E = 6 \cdot 10^{-2} \text{ J.}$$

### 2η λύση

$$\bullet \quad t = \frac{T}{12} = \frac{8\pi \cdot 10^{-3}}{12} = \frac{2\pi \cdot 10^{-3}}{3} \text{ s}$$

$$\bullet \quad E = \frac{1}{2} LI^2 \Rightarrow 8 \cdot 10^{-2} = \frac{1}{2} \cdot 16 \cdot 10^{-2} I^2 \Rightarrow I^2 = 1 \Rightarrow I = 1 \text{ A}$$

$$\bullet \quad \text{και } \omega = \frac{2\pi}{T} = \frac{2\pi}{8\pi \cdot 10^{-3}} = \frac{10^3}{4} = 250 \text{ rad/s}$$

$$\text{Άρα, } i = -I \eta \mu \omega t = -1 \cdot \eta \mu 250 \cdot \frac{2\pi \cdot 10^{-3}}{3} \Rightarrow$$

$$\Rightarrow i = -\eta \mu \frac{0,5\pi}{3} = -\eta \mu \frac{\pi}{6} = -\frac{1}{2} \text{ A}$$

$$\text{Άρα } U_E = 8 \cdot 10^{-2} \left[ 1 - \left(-\frac{1}{2}\right)^2 \right] = 8 \cdot 10^{-2} \left( 1 - \frac{1}{4} \right) = 8 \cdot 10^{-2} \cdot \frac{3}{4} = 6 \cdot 10^{-2} \text{ J.}$$

$$\Gamma 3. \quad E_{\text{AYT}} = -L \frac{di}{dt} \Rightarrow \frac{di}{dt} = \frac{-E_{\text{AYT}}}{L} = \frac{-v_c}{L} = \frac{q}{LC} = -\frac{q}{LC}$$

$$\text{Άρα } \frac{di}{dt} = -\omega^2 q$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{8\pi \cdot 10^{-3}} \Rightarrow \omega = 250 \text{ rad/s}$$

$$\left. \begin{array}{l} E = U_E + U_B \\ U_E = 3U_B \end{array} \right\} E = \frac{4}{3} U_E \Rightarrow \frac{Q^2}{2C} = \frac{4}{3} \frac{q^2}{2C}$$

$$\Rightarrow q = \frac{\pm\sqrt{3}}{2} Q$$

$$\text{Όμως } Q = CV = 10^{-4} \cdot 40 \Rightarrow Q = 4 \cdot 10^{-3} \text{ C}$$

$$q = \frac{\pm\sqrt{3}}{2} 4 \cdot 10^{-3} \Rightarrow q = \pm 2\sqrt{3} \cdot 10^{-3} \text{ C}$$

$$\text{άρα } \left| \frac{di}{dt} \right| = |-\omega^2 q| = 250^2 \cdot 2\sqrt{3} \cdot 10^{-3} \Rightarrow \frac{di}{dt} = 125 \cdot \sqrt{3} \text{ A/s}$$

**2η λύση:** για απόδειξη.  $\frac{\Delta i}{\Delta t}$ :

Σε απόλυτη αντιστοιχία μηχανικής και ηλεκτρικής ταλάντωσης ισχύει:

$$\frac{\Delta v}{\Delta t} = a = -\omega^2 x, \text{ άρα αντίστοιχα}$$

$$\frac{\Delta i}{\Delta t} = -\omega^2 q.$$

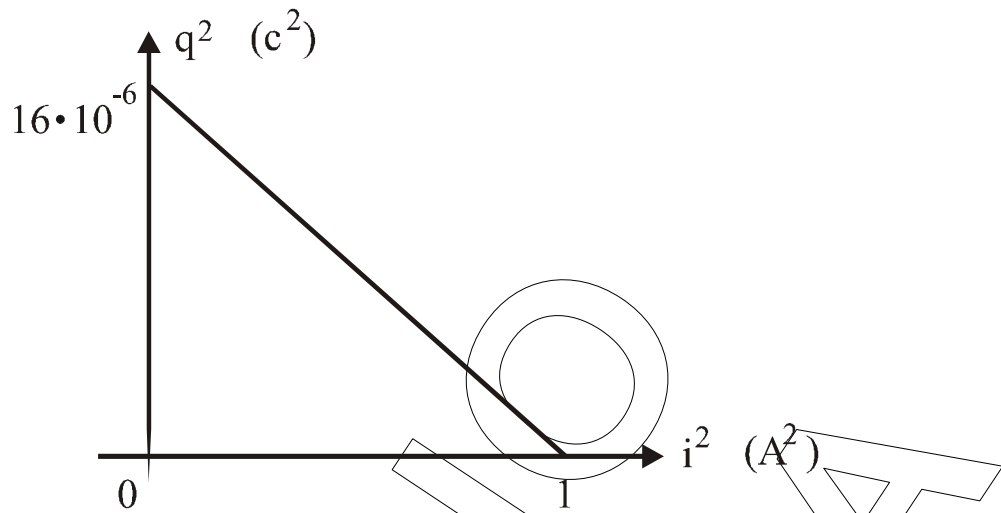
**Γ4.**

$$E = U_E + U_B \Rightarrow 8 \cdot 10^{-2} = \frac{q^2}{2C} + \frac{Li^2}{2} \Rightarrow$$

$$\Rightarrow 8 \cdot 10^{-2} = \frac{q^2}{2 \cdot 10^{-4}} + 8 \cdot 10^{-2} \cdot i^2 \Rightarrow \frac{q^2}{2 \cdot 10^{-4}} = 8 \cdot 10^{-2} - 8 \cdot 10^{-2} \cdot i^2$$

$$\text{άρα } q^2 = 16 \cdot 10^{-6} - 16 \cdot 10^{-6} \cdot i^2$$

$$\text{με } I = \omega \cdot Q = 250 \cdot 4 \cdot 10^{-3} \Rightarrow I = 1 \text{ A}$$

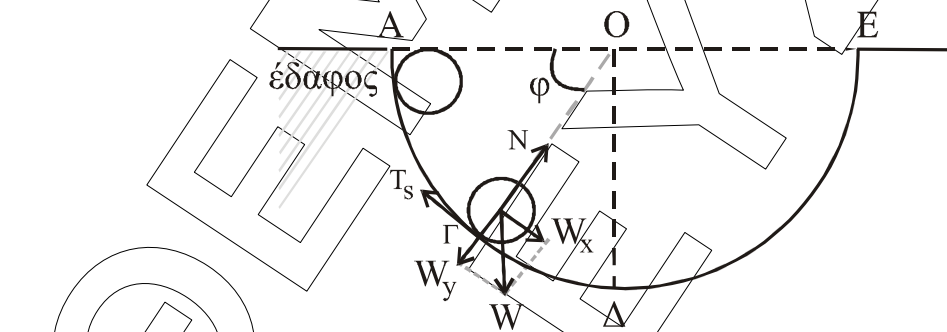


**ΘΕΜΑ Δ**

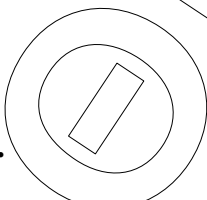
$R = 1,6 \text{ m}$

$m = 1,4 \text{ Kg}$

$r = R/8$



**Δ1.**



$\Sigma \tau = I_{\alpha \gamma} \Rightarrow$

$T_s \cdot r = \frac{2}{5} m r^2 \alpha_\gamma \Rightarrow$  Όμως  $\alpha_{cm} = \alpha_\gamma \cdot r$  (3)

$T_s = \frac{2}{5} m r \alpha_\gamma$  (1)

$\Sigma F_x = m a_{cm} \Rightarrow$

$W_x - T_s = m a_{cm} \Rightarrow$

$m g \sin \phi - T_s = m a_{cm}$  (2)

$$\text{Από (1) + (2) + (3) } mg\sigma\upsilon\upsilon\varphi = \frac{2}{5}ma_{\text{cm}} + ma_{\text{cm}} \Rightarrow$$

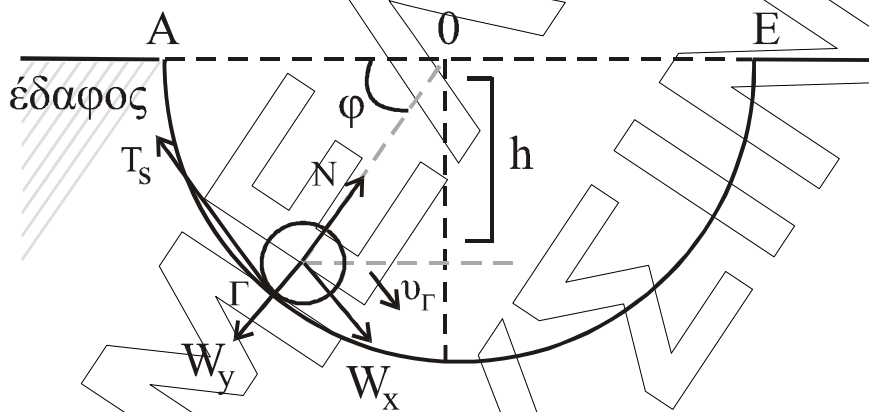
$$mg\sigma\upsilon\upsilon\varphi = \frac{7}{5}ma_{\text{cm}} \Rightarrow$$

$$a_{\text{cm}} = \frac{5}{7}g\sigma\upsilon\upsilon\varphi$$

$$T_s = \frac{2}{5}ma_{\text{cm}} = \frac{2}{5}m \frac{5}{7}g\sigma\upsilon\upsilon\varphi = \frac{10}{35}mg\sigma\upsilon\upsilon\varphi \Rightarrow$$

$$T_s = \frac{2}{7}mg\sigma\upsilon\upsilon\varphi = \frac{2}{7}1,4 \cdot 10 \cdot \sigma\upsilon\upsilon\varphi = 4 \cdot \sigma\upsilon\upsilon\varphi$$

Δ2.



$$\Sigma F_y = F_k \Rightarrow N - W_y = m \frac{v_{\Gamma}^2}{R-r} \Rightarrow N = mg\eta\mu\varphi + m \frac{v_{\Gamma}^2}{R-r} \quad (1)$$

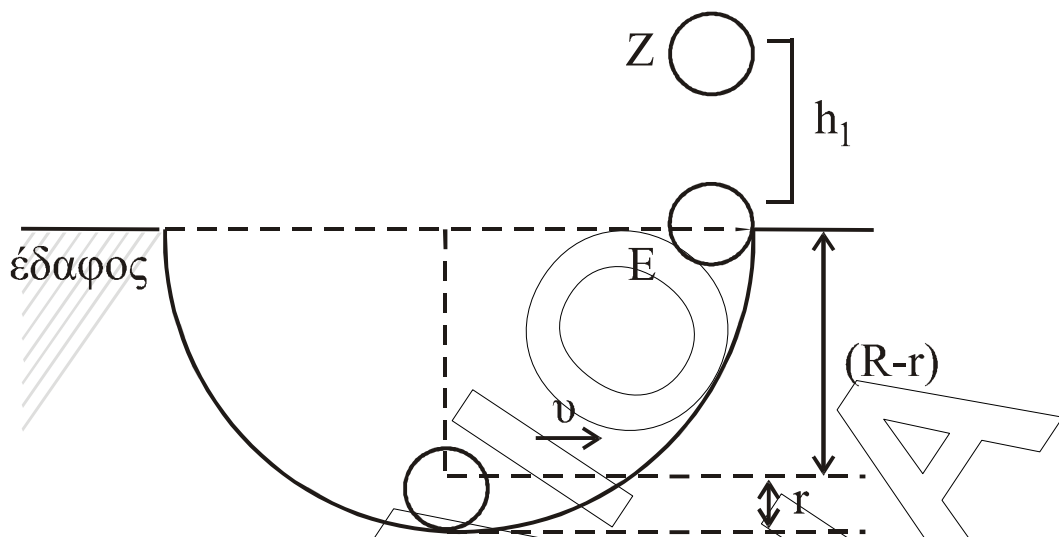
$$\text{ΑΔΜΕ } A \rightarrow \Gamma \quad E_{\mu\eta\chi(A)} = E_{\mu\eta\chi(\Gamma)} \Rightarrow mgh = \frac{1}{2}mv_{\Gamma}^2 + \frac{1}{2}I\omega_{\Gamma}^2 \Rightarrow$$

$$\Rightarrow mg(R-r)\eta\mu\varphi = \frac{1}{2}mv_{\Gamma}^2 + \frac{1}{2} \frac{2}{5}mr^2\omega_{\Gamma}^2 \Rightarrow g(R-r)\eta\mu\varphi = \frac{1}{2}v_{\Gamma}^2 + \frac{1}{5}v_{\Gamma}^2 \Rightarrow$$

$$\Rightarrow g(R-r)\eta\mu\varphi = \frac{7}{10}v_{\Gamma}^2 \Rightarrow v_{\Gamma}^2 = \frac{5}{7}g(R-r)$$

$$\text{Άρα (1) } N = mg\eta\mu\varphi + mg \frac{5}{7} = 1,4 \cdot 10 \cdot \frac{1}{2} + 1,4 \cdot 10 \cdot \frac{5}{7} \Rightarrow N = 17\text{N.}$$

Δ3.



Όμως  $v = v_{cm} = \omega \cdot r$

$\Delta \rightarrow E$

$$E_{\mu\eta\chi(\Delta)} = E_{\mu\eta\chi(E)} \Rightarrow \frac{1}{2} \cdot m \cdot v^2 + \frac{1}{2} \cdot I \cdot \omega^2 = \frac{1}{2} \cdot m \cdot v_E^2 + \frac{1}{2} \cdot I \cdot \omega_E^2 + mg(R-r) \Rightarrow$$

$$\Rightarrow \frac{1}{2} \cdot m \cdot v^2 + \frac{1}{2} \cdot \frac{2}{5} \cdot m \cdot r^2 \cdot \omega^2 = \frac{1}{2} \cdot m \cdot v_E^2 + \frac{1}{2} \cdot \frac{2}{5} \cdot m \cdot r^2 \cdot \omega_E^2 + mg(R-r) \Rightarrow$$

$$\Rightarrow \frac{1}{2} \cdot v^2 + \frac{1}{5} \cdot v^2 = \frac{1}{2} \cdot v_E^2 + \frac{1}{5} \cdot v_E^2 + g \cdot (R-r) \Rightarrow$$

$$\Rightarrow \frac{7}{10} \cdot v^2 = \frac{7}{10} \cdot v_E^2 + g \cdot \left( R - \frac{R}{8} \right) \Rightarrow \frac{7}{10} \cdot v_E^2 = \frac{7}{10} \cdot v^2 - g \cdot \frac{7R}{8} \Rightarrow$$

$$\Rightarrow \frac{7}{10} \cdot v_E^2 = \frac{7}{10} \cdot 6^2 - 10 \cdot \frac{7}{8} \cdot 1,6 \Rightarrow v_E^2 = 36 - \frac{10 \cdot 14}{7} = 16 \Rightarrow v_E = 4 \text{ m/s}$$

**ΑΔΜΕ**

$(E) \rightarrow (Z): E_{\mu\eta\chi(E)} = E_{\mu\eta\chi(Z)} \Rightarrow \frac{1}{2} \cdot I \cdot \omega_E^2 + \frac{1}{2} \cdot m \cdot v_E^2 = m \cdot g \cdot h_1 + \frac{1}{2} \cdot I \cdot \omega_Z^2 \Rightarrow$

$$\Rightarrow h_1 = \frac{v_E^2}{2g} = \frac{16}{2 \cdot 10} = 0,8 \text{ m.}$$

Άρα σταματά/στιγμαία 0,8m από την επιφάνεια του εδάφους.

Δ4. Στο E:  $\frac{\Delta K}{\Delta t} = -m \cdot g \cdot v_E = -1,4 \cdot 10 \cdot 4 = -56 \text{ J/s}$

$$\frac{\Delta L}{\Delta t} = \tau_w = 0$$